Students leaving Geometry or Geometry Honors and choosing to enroll in Algebra 2 Honors are required to have an understanding of these skills. It will be assumed that students can complete problems similar to those in this packet upon arriving on the first day of class. Although the practice problems will not be checked for a grade, these skills will be assessed by a test during the first week of class. This test will assess the students' understanding of the 8 topics in this packet. Use the packet as a resource to review and practice these 8 topics.

**Purpose:** The purpose of this assignment is to help students and their parents determine whether or not they are prepared for Algebra 2 Honors. Students’ test results will be reported to the students within the first week of the semester.

**Outcome:** The Algebra 2 Honors teacher of any student who receives a D (69%) or lower on the test will contact the student’s parent and guidance counselor to discuss either an alternate course selection or what actions the student would need to take to stay in the course and be successful.

**Help:** Students working on this packet who require help can email Miss Cubillan at rcubillan@tvsd.org. Emails over the summer will be answered once per week. Emails during the school year will be answered more regularly. If you need another copy of this packet, go to the library website. You can also stop by the high school office to pick up a copy if you do not have internet access.

**Algebra II Honors Prerequisite Skills to be Tested:**

- Graphing Lines
- Solving equations with variable on both sides
- Quadratic Formula
- Simplifying Quadratics by: factoring, zero product property, and multiplying binomials
- Solving Systems of Equations
- Right Triangle Trig
- Similar Triangles
- Laws of Positive Exponents
WRITING AND GRAPHING LINEAR EQUATIONS ON A FLAT SURFACE

**SLOPE** is a number that indicates the steepness (or flatness) of a line, as well as its direction (up or down) left to right.

**SLOPE** is determined by the ratio: \( \frac{\text{vertical change}}{\text{horizontal change}} \) between any two points on a line.

For lines that go up (from left to right), the sign of the slope is positive. For lines that go down (left to right), the sign of the slope is negative.

Any linear equation written as \( y = mx + b \), where \( m \) and \( b \) are any real numbers, is said to be in **SLOPE-INTERCEPT FORM**. \( m \) is the **SLOPE** of the line. \( b \) is the **Y-INTERCEPT**, that is, the point \((0, b)\) where the line intersects (crosses) the y-axis.

If two lines have the same slope, then they are parallel. Likewise, **PARALLEL LINES** have the same slope.

Two lines are **PERPENDICULAR** if the slope of one line is the negative reciprocal of the slope of the other line, that is, \( m \) and \( -\frac{1}{m} \). Note that \( m \cdot \left( -\frac{1}{m} \right) = -1 \).

Examples: \( 3 \) and \( -\frac{1}{3} \), \( -\frac{2}{3} \) and \( \frac{3}{4} \), \( \frac{5}{4} \) and \( -\frac{4}{5} \)

Two distinct lines on a flat surface that are not parallel intersect in a single point.
Example 1

Graph the linear equation \( y = \frac{4}{7} x + 2 \)

Using \( y = mx + b \), the slope is \( \frac{4}{7} \) and the y-intercept is the point \((0, 2)\). To graph, begin at the y-intercept \((0, 2)\). Remember that slope is \( \frac{\text{vertical change}}{\text{horizontal change}} \) so go up 4 units (since 4 is positive) from \((0, 2)\) and then move right 7 units. This gives a second point on the graph. To create the graph, draw a straight line through the two points.

Example 2

A line has a slope of \( \frac{3}{4} \) and passes through \((3, 2)\). What is the equation of the line?

Using \( y = mx + b \), write \( y = \frac{3}{4} x + b \). Since \((3, 2)\) represents a point \((x, y)\) on the line, substitute 3 for \( x \) and 2 for \( y \), \( 2 = \frac{3}{4} (3) + b \), and solve for \( b \).

\[ 2 = \frac{9}{4} + b \Rightarrow 2 - \frac{9}{4} = b \Rightarrow -\frac{1}{4} = b \]. The equation is \( y = \frac{3}{4} x - \frac{1}{4} \).
Example 3

Find two equations of the line through the given point, one parallel and one perpendicular to the given line: \( y = -\frac{5}{2} x + 5 \) and \((-4, 5)\).

<table>
<thead>
<tr>
<th>For the parallel line, use ( y = mx + b ) with the same slope to write ( y = -\frac{5}{2} x + b ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Substitute the point ((-4, 5)) for ( x ) and ( y ) and solve for ( b ).</td>
</tr>
<tr>
<td>[ 5 = -\frac{5}{2} (-4) + b \implies 5 = \frac{20}{2} + b \implies -5 = b ]</td>
</tr>
<tr>
<td>Therefore the parallel line through ((-4, 5)) is ( y = -\frac{5}{2} x - 5 ).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>For the perpendicular line, use ( y = mx + b ) where ( m ) is the negative reciprocal of the slope of the original equation to write ( y = \frac{2}{5} x + b ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Substitute the point ((-4, 5)) and solve for ( b ).</td>
</tr>
<tr>
<td>[ 5 = \frac{2}{5} (-4) + b \implies \frac{33}{5} = b ]</td>
</tr>
<tr>
<td>Therefore the perpendicular line through ((-4, 5)) is ( y = \frac{2}{5} x + \frac{33}{5} ).</td>
</tr>
</tbody>
</table>
Graph each line.

1. \( y = \frac{1}{2}x - 2 \)  
2. \( y = -\frac{3}{5}x - \frac{5}{3} \)  
3. \( 3x + 2y = 12 \)
4. \( x - y = -13 \)  
5. \( 2x - 4y = 12 \)  
6. \( 4y - 2x = 12 \)

Write the equation of the line with:

7. slope = \( \frac{1}{2} \) and passing through (4, 3).  
8. slope = \( \frac{2}{3} \) and passing through (-3, -2).
9. slope = \( -\frac{1}{3} \) and passing through (4, -1).  
10. slope = -4 and passing through (-3, 5).

Using the slope and y-intercept, determine the equation of the line.

14.  
15.  
16.  
17.

Find an equation of the line through the given point and parallel to the given line.

31. \( y = 2x - 2 \) and (-3, 5)  
32. \( y = \frac{1}{2}x + 3 \) and (-4, 2)
33. \( x - y = 2 \) and (-2, 3)  
34. \( y - x = -1 \) and (-2, 1)
35. \( x + 3y = 6 \) and (-1, 1)  
36. \( 3x + 2y = 6 \) and (2, -1)
Find an equation of the line through the given point and perpendicular to the given line.

39. \( y = 2x - 2 \) and \((-3, 5)\)  
40. \( y = \frac{1}{2} x + 3 \) and \((-4, 2)\)

41. \( x - y = 2 \) and \((-2, 3)\)  
42. \( y - x = -1 \) and \((-2, 1)\)

43. \( x + 3y = 6 \) and \((-1, 1)\)  
44. \( 3x + 2y = 6 \) and \((2, -1)\)

49. Write the equation of the line through \((7, -8)\) which is parallel to the line through \((2, 5)\) and \((8, -3)\)

50. Write the equation of the line through \((1, -4)\) which is parallel to the line through \((-3, -7)\) and \((4, 3)\)
Answers

1. (0, -2)  
2. \((0, -\frac{5}{3})\)  
3. (0, 6)  
4. (0, 13)

5. (0, -3)  
6. (0, 3)  
7. \(y = \frac{1}{2} x + 1\)  
8. \(y = \frac{2}{3} x\)

9. \(y = -\frac{1}{3} x + \frac{1}{3}\)  
10. \(y = -4x - 7\)

14. \(y = 2x - 2\)  
15. \(y = -x + 2\)  
16. \(y = \frac{1}{3} x + 2\)

17. \(y = -2x + 4\)

31. \(y = 2x + 11\)  
32. \(y = \frac{1}{2} x + 4\)  
33. \(y = x + 5\)  
34. \(y = x + 3\)

35. \(y = -\frac{1}{3} x + \frac{2}{3}\)  
36. \(y = -\frac{3}{2} x + 2\)

39. \(y = -\frac{1}{2} x + \frac{7}{2}\)  
40. \(y = -2x - 6\)  
41. \(y = -x + 1\)  
42. \(y = -x - 1\)

43. \(y = 3x + 4\)  
44. \(y = \frac{2}{3} x - \frac{7}{3}\)

49. \(y = -\frac{4}{3} x + \frac{4}{3}\)  
50. \(y = \frac{10}{7} x - \frac{38}{7}\)
SOLVING LINEAR EQUATIONS

Solving equations involves “undoing” what has been done to create the equation. In this sense, solving an equation can be described as “working backward,” generally known as using inverse (or opposite) operations. For example, to undo $x + 2 = 5$, that is, adding 2 to $x$, subtract 2 from both sides of the equation. The result is $x = 3$, which makes $x + 2 = 5$ true. For $2x = 17$, $x$ is multiplied by 2, so divide both sides by 2 and the result is $x = 8.5$. For equations like those in the examples and exercises below, apply the idea of inverse (opposite) operations several times. Always follow the correct order of operations.
**Example 1**

Solve for $x$: $2(2x - 1) - 6 = -x + 2$

First distribute to remove parentheses, then combine like terms.

$4x - 2 - 6 = -x + 2$

$4x - 8 = -x + 2$

Next, move variables and constants by addition of opposites to get the variable term on one side of the equation.

$4x - 8 = -x + 2$

$+x + x$

$5x - 8 = 2$

$5x - 8 = 2$

$+8 +8$

$5x = 10$

Now, divide by 5 to get the value of $x$.

$\frac{5x}{5} = \frac{10}{5} \Rightarrow x = 2$

Finally, check that your answer is correct.

**Example 2**

Solve for $y$: $2x + 3y - 9 = 0$

This equation has two variables, but the instruction says to isolate $y$. First move the terms without $y$ to the other side of the equation by adding their opposites to both sides of the equation.

$2x + 3y - 9 = 0$

$+9 +9$

$2x + 3y = +9$

$2x + 3y = 9$

$-2x \quad -2x$

$3y = -2x + 9$

Divide by 3 to isolate $y$. Be careful to divide every term on the right by 3.

$\frac{3y}{3} = \frac{-2x + 9}{3} \Rightarrow y = -\frac{2}{3}x + 3$

Finally, check that your answer is correct.
Solve each equation below.

1. \(5x + 2 = -x + 14\)  
2. \(3x - 2 = x + 10\)  
3. \(6x + 4x - 2 = 15\)
4. \(\frac{3}{4}x + 2 = -7\)
5. \(3(2x + 2) + 2(x - 7) = x + 3\)
6. \(2(x + 3) + 5(x - 2) = -x + 10\)
7. \(4 - 6(w + 2) = 10\)

Solve for the named variable.

17. \(3x + 2y = 6\) (for \(y\))
18. \(-3x + 5y = -10\) (for \(y\))
19. \(y = mx + b\) (for \(b\))
20. \(y = mx + b\) (for \(x\))
Answers

1. 2
2. 6
3. \( \frac{17}{10} \)
6. -12
9. \( \frac{11}{7} \)
10. \( \frac{7}{4} \)
11. -3
17. \( y = -\frac{3}{2}x + 3 \)
18. \( y = \frac{3}{5}x - 2 \)
19. \( b = y - mx \)
20. \( x = \frac{y-b}{m} \)
THE QUADRATIC FORMULA

You have used factoring and the Zero Product Property to solve quadratic equations. You can solve any quadratic equation by using the QUADRATIC FORMULA.

\[
\text{If } ax^2 + bx + c = 0, \quad \text{then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.
\]

For example, suppose \(3x^2 + 7x - 6 = 0\). Here \(a = 3\), \(b = 7\), and \(c = -6\).

Substituting these values into the formula results in:

\[
x = \frac{-7 \pm \sqrt{7^2 - 4(3)(-6)}}{2(3)} \Rightarrow x = \frac{-7 \pm \sqrt{121}}{6} \Rightarrow x = \frac{-7 \pm 11}{6}
\]

Remember that non-negative numbers have both a positive and negative square root. The sign \(\pm\) represents this fact for the square root in the formula and allows us to write the equation once (representing two possible solutions) until later in the solution process.

Split the numerator into the two values:

\[
x = \frac{-7 + 11}{6} \quad \text{or} \quad x = \frac{-7 - 11}{6}
\]

Thus the solution for the quadratic equation is:

\[
x = \frac{2}{3} \quad \text{or} \quad -3.
\]

Use the quadratic formula to solve each of the following equations.

1. \(x^2 - x - 6 = 0\)  
   2. \(x^2 + 8x + 15 = 0\)

3. \(x^2 + 13x + 42 = 0\)  
   4. \(x^2 - 10x + 16 = 0\)

13. \(4x^2 - 9x + 4 = 0\)  
   14. \(2x^2 - 5x + 2 = 0\)

15. \(20x^2 + 20x = 1\)  
   16. \(13x^2 - 16x = 4\)

17. \(7x^2 + 28x = 0\)  
   18. \(5x^2 = -125x\)

19. \(8x^2 - 50 = 0\)  
   20. \(15x^2 = 3\)
Answers

1. \( x = -2, 3 \)  
2. \( x = -5, -3 \)  
3. \( x = -7, -6 \)  
4. \( x = 2, 8 \)

13. \( x = \frac{9 \pm \sqrt{7}}{8} \)  
14. \( x = 2, \frac{1}{2} \)  
15. \( x = \frac{-20 \pm \sqrt{480}}{40} = \frac{-5 \pm \sqrt{30}}{10} \)

16. \( x = \frac{16 \pm \sqrt{464}}{26} = \frac{8 \pm 2\sqrt{29}}{13} \)  
17. \( x = -4, 0 \)  
18. \( x = -25, 0 \)

19. \( x = -\frac{5}{2}, \frac{5}{2} \)  
20. \( x = \frac{\pm \sqrt{5}}{5} \)
FACTORING POLYNOMIALS

Often we want to un-multiply or FACTOR a polynomial \( P(x) \). This process involves finding a constant and/or another polynomial that evenly divides the given polynomial. In formal mathematical terms, this means \( P(x) = q(x) \cdot r(x) \), where \( q \) and \( r \) are also polynomials. For elementary algebra there are three general types of factoring.

1) **Common term** (finding the largest common factor):

\[
6x + 18 = 6(x + 3) \quad \text{where 6 is a common factor of both terms.}
\]
\[
2x^3 - 8x^2 - 10x = 2x(x^2 - 4x - 5) \quad \text{where } 2x \text{ is the common factor.}
\]
\[
2x^2(x - 1) + 7(x - 1) = (x - 1)(2x^2 + 7) \quad \text{where } x - 1 \text{ is the common factor.}
\]

2) **Special products**

\[
a^2 - b^2 = (a + b)(a - b) \quad x^2 - 25 = (x + 5)(x - 5)
\]
\[
9x^2 - 4y^2 = (3x + 2y)(3x - 2y)
\]
\[
x^2 + 2xy + y^2 = (x + y)^2 \quad x^2 + 8x + 16 = (x + 4)^2
\]
\[
x^2 - 2xy + y^2 = (x - y)^2 \quad x^2 - 8x + 16 = (x - 4)^2
\]

3a) **Trinomials** in the form \( x^2 + bx + c \) where the coefficient of \( x^2 \) is 1.

Consider \( x^2 + (d + e)x + d \cdot e = (x + d)(x + e) \), where the coefficient of \( x \) is the sum of two numbers \( d \) and \( e \) AND the constant is the product of the same two numbers, \( d \) and \( e \). A quick way to determine all of the possible pairs of integers \( d \) and \( e \) is to factor the constant in the original trinomial. For example, 12 is 1 \cdot 12, 2 \cdot 6, and 3 \cdot 4. The signs of the two numbers are determined by the combination you need to get the sum. The "sum and product" approach to factoring trinomials is the same as solving a "Diamond Problem" in CPM's Algebra 1 course (see below).

\[
x^2 + 8x + 15 = (x + 3)(x + 5); \quad 3 + 5 = 8, \quad 3 \cdot 5 = 15
\]
\[
x^2 - 2x - 15 = (x - 5)(x + 3); \quad -5 + 3 = -2, \quad -5 \cdot 3 = -15
\]
\[
x^2 - 7x + 12 = (x - 3)(x - 4); \quad -3 + (-4) = -7, \quad (-3)(-4) = 12
\]
The sum and product approach can be shown visually using rectangles for an area model. The figure at far left below shows the "Diamond Problem" format for finding a sum and product. Here is how to use this method to factor \(x^2 + 6x + 8\).

\[
\begin{array}{c|c|c|c|c|c|c}
\text{Product} & 8 & 4 & \hline & \quad 4 \quad & \\
\text{Sum} & 2 & 6 & \hline & \quad 2x \quad & 8 \quad \\
\end{array}
\]

\[
\begin{array}{c|c|c|c|c|c|c|c}
\text{Product} & x^2 & 4x & \hline & \quad x + 4 \quad & \\
\text{Sum} & 2x & 8 & \hline & \quad 2x \quad & 8 \quad \\
\end{array}
\]

\[
(x + 4)(x + 2)
\]

>> Explanation and examples continue on the next page. >>

3b) Trinomials in the form \(ax^2 + bx + c\) where \(a \neq 1\).

Note that the upper value in the diamond is no longer the constant. Rather, it is the product of \(a\) and \(c\), that is, the coefficient of \(x^2\) and the constant.

\[
\begin{array}{c|c|c|c|c|c|c|c}
6 & ? & 6 & \hline & \quad 6 \quad & \\
7 & ? & 7 & \hline & \quad 1 \quad & 3 \quad \\
\end{array}
\]

\[
\begin{array}{c|c|c|c|c|c|c|c}
2x^2 & 6x & \hline & \quad 2x \quad & 6 \quad \\
1x & 3 & \hline & \quad 1 \quad & 3 \quad \\
\end{array}
\]

\[
(2x + 1)(x + 3)
\]

Below is the process to factor \(5x^2 - 13x + 6\).

\[
\begin{array}{c|c|c|c|c|c|c|c}
30 & ? & 5x^2 & \hline & \quad 5x \quad & \\
-13 & ? & ? & \hline & \quad -3 \quad & \\
\end{array}
\]

\[
\begin{array}{c|c|c|c|c|c|c|c}
5x^2 & -10x & \hline & \quad 5x \quad & -10x \quad \\
-3x & 6 & \hline & \quad -3 \quad & 6 \quad \\
\end{array}
\]

\[
(5x - 3)(x - 2)
\]

Polynomials with four or more terms are generally factored by grouping the terms and using one or more of the three procedures shown above. Note that polynomials are usually factored completely. In the second example in part (1) above, the trinomial also needs to be factored. Thus, the complete factorization of \(2x^3 - 8x^2 - 10x = 2x(x^2 - 4x - 5) = 2x(x - 5)(x + 1)\).

Factor each polynomial completely.

1. \(x^2 - x - 42\)
2. \(4x^2 - 18\)
3. \(2x^2 + 9x + 9\)
4. \(2x^2 + 3xy + y^2\)
5. \(6x^2 - x - 15\)
6. \(4x^2 - 25\)
7. \(x^2 - 28x + 196\)
8. \(7x^2 - 847\)
9. \(x^2 + 18x + 81\)
10. \(x^2 + 4x - 21\)
11. \(3x^2 + 21x\)
12. \(3x^2 - 20x - 32\)
Factor completely.

28. \( 75x^3 - 27x \)
29. \( 3x^3 - 12x^2 - 36x \)
30. \( 4x^3 - 44x^2 + 112x \)
31. \( 5y^2 - 125 \)
32. \( 3x^2y^2 - xy^2 - 4y^2 \)
33. \( x^3 + 10x^2 - 24x \)
Factor each of the following completely. Use the modified diamond approach.

37. \(2x^2 + 5x - 7\)  
38. \(3x^2 - 13x + 4\)  
39. \(2x^2 + 9x + 10\)  
40. \(4x^2 - 13x + 3\)  
41. \(4x^2 + 12x + 5\)  
42. \(6x^3 + 31x^2 + 5x\)
Answers

1. \((x + 6)(x - 7)\)  
2. \(2\left(2x^2 - 9\right)\)  
3. \((2x + 3)(x + 3)\)

4. \((2x + y)(x + y)\)  
5. \((2x + 3)(3x - 5)\)  
6. \((2x - 5)(2x + 5)\)

7. \((x - 14)^2\)  
8. \(7(x - 11)(x + 11)\)  
9. \((x + 9)^2\)

10. \((x + 7)(x - 3)\)  
11. \(3x(x + 7)\)  
12. \((x - 8)(3x + 4)\)

27. \((x + 8)^2\)

28. \(3x(5x - 3)(5x + 3)\)  
29. \(3x(x - 6)(x + 2)\)  
30. \(4x(x - 7)(x - 4)\)

31. \(5(y + 5)(y - 5)\)  
32. \(y^2(3x - 4)(x + 1)\)  
33. \(x(x + 12)(x - 2)\)

34. \(3x(x - 5)(x + 3)\)

37. \((2x + 7)(x - 1)\)  
38. \((3x - 1)(x - 4)\)  
39. \((x + 2)(2x + 5)\)

40. \((4x - 1)(x - 3)\)  
41. \((2x + 5)(2x + 1)\)  
42. \(x(6x + 1)(x + 5)\)
If $a \cdot b = 0$, then either $a = 0$ or $b = 0$.

Note that this property states that at least one of the factors MUST be zero. It is also possible that all of the factors are zero. This simple statement gives us a powerful result which is most often used with equations involving the products of binomials. For example, solve $(x + 5)(x - 2) = 0$.

By the Zero Product Property, since $(x + 5)(x - 2) = 0$, either $x + 5 = 0$ or $x - 2 = 0$. Thus, $x = -5$ or $x = 2$.

The Zero Product Property can be used to find where a quadratic crosses the x-axis. These points are the x-intercepts. In the example above, they would be $(-5, 0)$ and $(2, 0)$.

Use factoring and the Zero Product Property to find the x-intercepts of each parabola below. Express your answer as ordered pair(s).

9. $y = x^2 - 3x + 2$  
10. $y = x^2 - 10x + 25$  
11. $y = x^2 - x - 12$  
12. $y = x^2 - 4x - 5$
13. $y = x^2 + 2x - 8$  
14. $y = x^2 + 6x + 9$  
15. $y = x^2 - 8x + 16$  
16. $y = x^2 - 9$
Answers

9.  (1, 0) and (2, 0)

10. (5, 0)  11. (–3, 0) and (4, 0)  12. (5, 0) and (–1, 0)

13. (–4, 0) and (2, 0)  14. (–3, 0)  15. (4, 0)

16. (3, 0) and (–3, 0)
SOLVING LINEAR SYSTEMS

You can find where two lines intersect (cross) by using algebraic methods. The two most common methods are **SUBSTITUTION** and **ELIMINATION** (also known as the addition method).

Example 1

**Substitution** can also be used when the equations are **not** in y-form.

Use substitution to rewrite the two equations as

\[
4(-3y + 1) - 3y = -11
\]

by replacing \(x\) with \((-3y + 1)\), then solve for \(y\) as shown at right.

Substitute \(y = 1\) into \(x = -3y + 1\). Solve for \(x\), and write the answer for \(x\) and \(y\) as an ordered pair, \((1, -2)\). Substitute \(y = 1\) into \(4x - 3y = -11\) to verify that **either** original equation may be used to find the second coordinate.
Example 2

When you have a pair of two-variable equations, sometimes it is easier to **ELIMINATE** one of the variables to obtain one single variable equation. You can do this by adding the two equations together as shown in the example below.

Solve the system at right:

\[
\begin{align*}
2x + y &= 11 \\
x - y &= 4 \\
\end{align*}
\]

To eliminate the \( y \) terms, **add** the two equations together.

\[
\begin{align*}
2x + y &= 11 \\
x - y &= 4 \\
\hline
3x &= 15 \\
\end{align*}
\]

then solve for \( x \).

\[
3x = 15 \\
\Rightarrow x = 5 \\
\]

Once we know the \( x \)-value we can substitute it into either of the original equations to find the corresponding value of \( y \).

Using the first equation:

\[
\begin{align*}
2x + y &= 11 \\
2(5) + y &= 11 \\
10 + y &= 11 \\
\Rightarrow y &= 1 \\
\end{align*}
\]

Example 3

You can solve the system of equations at right by elimination, but before you can eliminate one of the variables, you must adjust the coefficients of one of the variables so that they are additive opposites.

\[
\begin{align*}
3x + 2y &= 11 \\
4x + 3y &= 14 \\
\Rightarrow 9x + 6y &= 33 \\
-8x - 6y &= -28 \\
\end{align*}
\]

To eliminate \( y \), multiply the first equation by 3, then multiply the second equation by \(-2\) to get the equations at right.

Next eliminate the \( y \) terms by adding the two adjusted equations.

\[
\begin{align*}
9x + 6y &= 33 \\
-8x - 6y &= -28 \\
\hline
x &= 5 \\
\end{align*}
\]

Since \( x = 5 \), substitute in either original equation to find that \( y = -2 \). Therefore, the solution to the system of equations is \( (5, -2) \).

You could also solve the system by first multiplying the first equation by 4 and the second equation by \(-3\) to eliminate \( x \), then proceeding as shown above to find \( y \).
Solve the following systems of equations to find the point of intersection \((x, y)\) for each pair of lines.

1. \(y = x - 6\)  
   \(y = 12 - x\)

3. \(x = 7 + 3y\)  
   \(x = 4y + 5\)

8. \(x = \frac{1}{2}y + 4\)  
   \(8x + 3y = 31\)

9. \(2y = 4x + 10\)  
   \(6x + 2y = 10\)

13. \(x + y = 12\)  
    \(x - y = 4\)

14. \(2x - y = 6\)  
    \(4x - y = 12\)

15. \(x + 2y = 7\)  
    \(5x - 4y = 14\)

16. \(5x - 2y = 6\)  
    \(4x + y = 10\)

17. \(x + y = 10\)  
    \(x - 2y = 5\)

18. \(3y - 2x = 16\)  
    \(y = 2x + 4\)
Answers

1. (9, 3)  
3. (13, 2)  
8. $\left( \frac{7}{2}, 1 \right)$  
9. (0, 5)  
13. (8, 4)  
14. (3, 0)  
15. (4, 1.5)  
16. (2, 2)  
17. $\left( \frac{25}{3}, \frac{5}{3} \right)$  
18. (1, 6)
The three basic trigonometric ratios for right triangles are the sine (pronounced "sign"), cosine, and tangent. Each one is used in separate situations, and the easiest way to remember which to use when is the mnemonic SOH-CAH-TOA. With reference to one of the acute angles in a right triangle, Sine uses the Opposite and the Hypotenuse - SOH. The Cosine uses the Adjacent side and the Hypotenuse - CAH, and the Tangent uses the Opposite side and the Adjacent side -TOA. In each case, the position of the angle determines which leg (side) is opposite or adjacent. Remember that opposite means “across from” and adjacent means “next to.”

\[
\begin{align*}
\tan A &= \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{BC}{AC} \\
\sin A &= \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{BC}{AB} \\
\cos A &= \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{AC}{AB}
\end{align*}
\]
**Example 1**

Use trigonometric ratios to find the lengths of each of the missing sides of the triangle below.

The length of the adjacent side with respect to the 42° angle is 17 ft. To find the length y, use the tangent because y is the opposite side and we know the adjacent side.

\[
\tan 42° = \frac{y}{17}
\]

\[17 \tan 42° = y\]

\[15.307 \text{ ft} \approx y\]

The length of y is approximately 15.31 feet.

To find the length h, use the cosine ratio (adjacent and hypotenuse).

\[
\cos 42° = \frac{17}{h}
\]

\[h \cos 42° = 17\]

\[h = \frac{17}{\cos 42°} \approx 22.876 \text{ ft}\]

The hypotenuse is approximately 22.9 feet long.

**Example 2**

Use trigonometric ratios to find the size of each angle and the missing length in the triangle below.

To find m∠u, use the tangent ratio because you know the opposite (18 ft) and the adjacent (21 ft) sides.

\[
\tan u° = \frac{18}{21}
\]

\[m∠u° = \tan^{-1} \frac{18}{21} \approx 40.601°\]

The measure of angle u is approximately 40.6°. By subtraction we know that m∠v ≈ 49.4°.

Use the sine ratio for m∠u and the opposite side and hypotenuse.

\[
\sin 40.6° = \frac{18}{h}
\]

\[h \sin 40.6° = 18\]

\[h = \frac{18}{\sin 40.6°} \approx 27.659 \text{ ft}\]

The hypotenuse is approximately 27.7 feet long.
Use trigonometric ratios to solve for the variable in each figure below.

1. 

2. 

3. 

6. 

7. 

9. 

13. 

14. 

15. 

Draw a diagram and use trigonometric ratios to solve each of the following problems.

17. Juanito is flying a kite at the park and realizes that all 500 feet of string are out. Margie measures the angle of the string with the ground with her clinometer and finds it to be $42.5^\circ$. How high is Juanito’s kite above the ground?

18. Nell’s kite has a 350 foot string. When it is completely out, Ian measures the angle to be $47.5^\circ$. How far would Ian need to walk to be directly under the kite?
Answers

1. \( h = 15 \sin 38^\circ \approx 9.235 \)

2. \( h = 8 \sin 26^\circ \approx 3.507 \)

3. \( x = 23 \cos 49^\circ \approx 15.089 \)

6. \( y = 43 \tan 55^\circ \approx 61.4104 \)

7. \( z = \frac{15}{\sin 38^\circ} \approx 24.364 \)

9. \( w = \frac{23}{\cos 38^\circ} \approx 29.1874 \)

13. \( x = \tan^{-1} \frac{5}{7} \approx 35.5377^\circ \)

14. \( u = \tan^{-1} \frac{7}{9} \approx 37.875^\circ \)

15. \( y = \tan^{-1} \frac{12}{18} \approx 33.690^\circ \)

16. \( v = \tan^{-1} \frac{78}{88} \approx 41.5526^\circ \)

17. \[
\begin{align*}
\text{sin} 42.5^\circ &= \frac{h}{500} \\
h &= 500 \sin 42.5^\circ \approx 337.795 \text{ ft}
\end{align*}
\]

18. \[
\begin{align*}
\cos 47.5^\circ &= \frac{d}{350} \\
d &= 350 \cos 47.5^\circ \approx 236.46 \text{ ft}
\end{align*}
\]
**RATIO OF SIMILARITY**

The **RATIO OF SIMILARITY** between any two similar figures is the ratio of any pair of corresponding sides. Simply stated, once it is determined that two figures are similar, all of their pairs of corresponding sides have the same ratio.

The ratio of similarity of figure P to figure Q, written P : Q, is \( \frac{3}{5} \).

\( \frac{4.5}{7.5} \) and \( \frac{12}{20} \) have the same ratio.

Note that the ratio of similarity is always expressed in lowest possible terms. Also, the order of the statement, P : Q or Q : P, determines which order to state and use the ratios between pairs of corresponding sides.

An Equation that sets one ratio equal to another ratio is called a **proportion**. \( \frac{3}{5} = \frac{12}{20} \)

**Example**

Find \( x \) in the figure. Be consistent in matching corresponding parts of similar figures.

\( \triangle ABC \sim \triangle DEC \) by AA (\( \angle BAC \cong \angle EDC \) and \( \angle C \cong \angle C \)). Therefore the corresponding sides are proportional.

\[
\frac{DE}{AB} = \frac{CD}{CA} \implies \frac{22}{x} = \frac{24}{36} \implies 24x = 22(36) \implies 24x = 792 \implies x = 33
\]

Note: This problem also could have been solved with the proportion: \( \frac{DE}{AB} = \frac{CE}{CB} \).
For each pair of similar figures below, state the ratio of similarity, then use it to find $x$.

10.

11.

12.

For problems 13 through 18, use the given information and the figure to find each length.

13. $JM = 14$, $MK = 7$, $JN = 10$  
    Find $NL$.

14. $MN = 5$, $JN = 4$, $JL = 10$  
    Find $KL$.

15. $KL = 10$, $MK = 2$, $JM = 6$  
    Find $MN$.

16. $MN = 5$, $KL = 10$, $JN = 7$  
    Find $JL$.

17. $JN = 3$, $NL = 7$, $JM = 5$  
    Find $JK$.

18. $JK = 37$, $NL = 7$, $JM = 5$  
    Find $JN$.

19. Standing 4 feet from a mirror laying on the flat ground, Palmer, whose eye height is 5 feet, 9 inches, can see the reflection of the top of a tree. He measures the mirror to be 24 feet from the tree. How tall is the tree?

20. The shadow of a statue is 20 feet long, while the shadow of a student is 4 ft long. If the student is 6 ft tall, how tall is the statue?
**Answers**

10. \( \frac{5}{6} ; x = 15 \)

11. \( \frac{4}{5} ; x = 20 \)

12. \( \frac{3}{2} ; x = 16.5 \)

13. 5

14. 12.5

15. 7.5

16. 14

17. \( 16 \frac{2}{3} \)

18. \( 11 \frac{25}{32} \)

19. 34.5 ft.

20. 30 ft.
LAWS OF EXPONENTS

BASE, EXPONENT, AND VALUE

In the expression $2^5$, 2 is the **base**, 5 is the **exponent**, and the **value** is 32.

$2^5$ means $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$

$x^3$ means $x \cdot x \cdot x$

LAWS OF EXPONENTS

Here are the basic patterns with examples:

1) $x^a \cdot x^b = x^{a+b}$
   examples: $x^3 \cdot x^4 = x^{3+4} = x^7$; $2^7 \cdot 2^4 = 2^{11}$

2) $\frac{x^a}{x^b} = x^{a-b}$
   examples: $x^{10} \div x^4 = x^{10-4} = x^6$; $\frac{2^4}{2^7} = 2^{-3}$

3) $(x^a)^b = x^{ab}$
   examples: $(x^4)^3 = x^{4 \cdot 3} = x^{12}$; $(2x^3)^4 = 2^4 \cdot x^{12} = 16x^{12}$

4) $x^{-a} = \frac{1}{x^a}$ and $\frac{1}{x^{-b}} = x^b$
   examples: $3x^{-3}y^2 = \frac{3y^2}{x^3}$; $\frac{2x^5}{y^{-2}} = 2x^5y^2$
Simplify each expression:

1. $y^5 \cdot y^7$
2. $b^4 \cdot b^3 \cdot b^2$
3. $(y^5)^2$
4. $(3a)^4$
5. $\frac{m^8}{m^3}$
6. $\frac{12x^9}{4x^4}$
7. $\left(\frac{y^4}{y^3}\right)^2$
8. $(x^3y^2)^3$
9. $\left(\frac{15x^2y^7}{3x^4y^5}\right) (2y)^3$
10. $(4c^4)(ac^3)(3a^5c)$
11. $(7x^3y^5)^2$
12. $(4xy^2)(2y)^3$
13. $\left(\frac{4}{x^2}\right)^3$
14. $\left(\frac{2a^7}{3a^2}\right)$
15. $\left(\frac{5m^3n}{m^5}\right)^3$
16. $\left(\frac{3a^2x^3}{2ax^4}\right)$
17. $\left(\frac{x^3y}{y^4}\right)^4$
18. $\left(\frac{6y^2x^8}{12x^3y^7}\right)^2$
19. $\left(\frac{8x^7y^{12}}{2x^5y^3}(4xy^4)^2\right)$
Answers

1. \(y^{12}\)  
   2. \(b^{9}\)  
   4. \(y^{10}\)

5. \(81a^{4}\)  
   6. \(m^{5}\)  
   7. \(3x^{5}\)  
   8. \(x^{9}y^{6}\)

9. \(y^{2}\)  
   10. \(\frac{5y^{2}}{x^{2}}\)  
   11. \(12a^{6}c^{8}\)  
   12. \(49x^{6}y^{10}\)

13. \(32xy^{5}\)  
   14. \(\frac{64}{x^{6}}\)  
   15. \(a^{6}\)  
   16. \(\frac{125n^{3}}{m^{6}}\)

17. \(72a^{7}x^{18}\)  
   18. \(\frac{x^{12}}{y^{12}}\)  
   19. \(\frac{x^{10}}{4y^{10}}\)  
   20. \(16x^{10}y^{5}\)